

# GCSE Revision: Direct and Inverse Proportion



GCSE Tier: Higher

**ANSWERS**

Target Grade: 6-7

- 1)  $y$  is proportional to  $x^2$  when  $y = 18$  and  $x = 3$ .  
Find an equation for  $y$  in terms of  $x$ .

$$\begin{aligned}y &\propto x^2 \\y &= kx^2 \\18 &= k \times 3^2 \\18 &= k \times 9 \\k &= 2 \\y &= 2x^2\end{aligned}$$

- 2) When a stone is dropped, its speed,  $v$ , is proportional to the square root of the distance,  $d$ , it has fallen.  $v = 14 \text{ ms}^{-1}$  when  $d = 10 \text{ m}$   
(a) Find the formula for  $v$  in terms of  $d$ .

$$\begin{aligned}v &\propto \sqrt{d} \\v &= k\sqrt{d} \\14 &= k\sqrt{10} \\\frac{14}{\sqrt{10}} &= k \\v &= \frac{14}{\sqrt{10}}\sqrt{d} \\v &= 14\sqrt{\frac{d}{10}}\end{aligned}$$

- (b) Calculate the distance the stone has fallen when its speed is  $28 \text{ ms}^{-1}$

$$\begin{aligned}v &= 14\sqrt{\frac{d}{10}} \\28 &= 14\sqrt{\frac{d}{10}} \\\frac{28}{14} &= \sqrt{\frac{d}{10}} \\2 &= \sqrt{\frac{d}{10}} \\4 &= \frac{d}{10} \\d &= 40\end{aligned}$$

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3)  $S \propto \frac{1}{R}$  and  $S = 29.6$  when  $R = 4.5$ .

Find  $S$  when  $R = 7.2$ .

$$\begin{aligned} S &\propto \frac{1}{R} \\ S &= \frac{k}{R} \\ 29.6 &= \frac{k}{4.5} \\ 29.6 \times 4.5 &= k \\ k &= 133.2 \\ S &= \frac{133.2}{R} \\ S &= \frac{133.2}{7.2} \\ S &= 18.5 \end{aligned}$$

4)  $p$  is inversely proportional to  $q^3$  and  $p = 6$  when  $q = 2$ .  
Find an equation connecting  $p$  and  $q$ .

$$\begin{aligned} p &\propto \frac{1}{q^3} \\ p &= \frac{k}{q^3} \\ 6 &= \frac{k}{2^3} \\ 6 &= \frac{k}{8} \\ k &= 48 \\ p &= \frac{48}{q^3} \end{aligned}$$

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5) The distance,  $d$  km, it is possible to see on a clear day is proportional to the square root of the height,  $h$  m, above sea level. Standing on a pier, 4 m above sea level, it is possible to see a distance of 10 km.

(a) Find a formula for  $d$  in terms of  $h$ .

$$d \propto \sqrt{h}$$

$$d = k\sqrt{h}$$

$$10 = k\sqrt{4}$$

$$10 = k \times 2$$

$$k = 5$$

$$d = 5\sqrt{h}$$

Standing on top of the cliffs I can see a distance of 35 km.

(b) How high are the cliffs?

$$d = 5\sqrt{h}$$

$$35 = 5\sqrt{h}$$

$$7 = \sqrt{h}$$

$$h = 49$$

6)  $R$  is inversely proportional to  $d^2$  and  $R = 4$  when  $d = 3$ .

Express  $R$  in terms of  $d$ .

$$R \propto \frac{1}{d^2}$$

$$R = \frac{k}{d^2}$$

$$4 = \frac{k}{3^2}$$

$$4 = \frac{k}{9}$$

$$36 = k$$

$$R = \frac{36}{d^2}$$

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- 7) The wavelength of a radio wave is inversely proportional to its frequency.  
The wavelength is 750 m when the frequency is 400 kHz.  
Calculate the wavelength when the frequency is 320 kHz.

$$\begin{aligned}W &\propto \frac{1}{f} \\W &= \frac{k}{f} \\750 &= \frac{k}{400} \\750 \times 400 &= k \\k &= 300\,000 \\W &= \frac{300\,000}{f} \\W &= \frac{300\,000}{320} \\W &= 937.5 \text{ m}\end{aligned}$$

- 8) The time,  $t$  seconds, that a ball takes to roll from rest down a slope is proportional to the square root of the distance,  $d$  metres, that it rolls.  
It takes 4 seconds for the ball to roll 25 metres down the slope.  
(a) Find an expression for  $t$  in terms of  $d$ .

$$\begin{aligned}t &\propto \sqrt{d} \\t &= k\sqrt{d} \\4 &= k\sqrt{25} \\4 &= k \times 5 \\k &= 4 \div 5 \\k &= 0.8 \\t &= 0.8\sqrt{d}\end{aligned}$$

- (b) Find the time it takes for the ball to roll 64 metres down the slope.

$$\begin{aligned}t &= 0.8\sqrt{d} \\t &= 0.8\sqrt{64} \\t &= 0.8 \times 8 \\t &= 6.4 \text{ seconds}\end{aligned}$$

- (c) Find how far down the slope the ball will roll in 8 seconds.

$$\begin{aligned}t &= 0.8\sqrt{d} \\8 &= 0.8\sqrt{d} \\8 \div 0.8 &= \sqrt{d} \\10 &= \sqrt{d} \\d &= 10^2 \\d &= 100\end{aligned}$$

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9) When  $x$  takes a certain value, the value of  $y = 10$ .

If this value of  $x$  is multiplied by 4, work out the value of  $y$  when:

(a)  $y$  is proportional to  $x$

$$10 \times 4 = 40$$

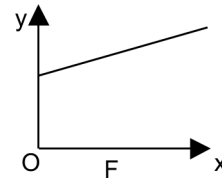
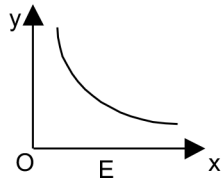
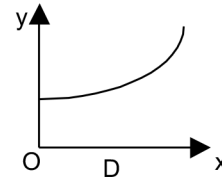
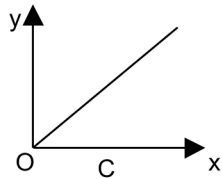
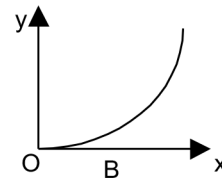
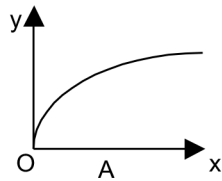
(b)  $y$  is proportional to  $x^2$

$$10 \times 16 = 160$$

(c)  $y$  is inversely proportional to  $x$

$$10 \times 1/4 = 2.5$$

10) Choose which of the graphs below best represents each of these relationships.



(a)  $y$  is proportional to  $x$ .

Graph C

(b)  $y$  is proportional to  $x^2$ .

Graph B

(c)  $y$  is inversely proportional to  $x$ .

Graph E

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11) In the table below,  $x$  and  $y$  are connected by an equation of the form  $y = k x^n$ .

$x$	2	4	6	$p$
$y$	12	48	108	147

Find the values of  $k$  and  $n$ .

$x$	2	4	6
$y$	12	48	108
$y/x$	6	12	18

Does not work.

Try  $n = 2$ :

$x^2$	4	16	36
$y$	12	48	108
$y/x$	3	3	3

So  $y = k x^2$

Substitute  $x$  &  $y$  into the formula:

$$\begin{aligned}y &= k x^2 \\12 &= k \times 2^2 \\12 &= k \times 4 \\k &= 3\end{aligned}$$

So  $y = 3 x^2$

12) Find an equation connecting  $x$  and  $y$  for the following table of values.

$x$	4	9	16	25
$y$	4	6	8	10

First notice that the values of  $x$  are all square numbers:

$x$	4	9	16	25
$y$	4	6	8	10

$\sqrt{x}$	2	3	4	5
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Now notice that the values of  $y$  are two times the values of  $x$ :

$x$	4	9	16	25
$y$	4	6	8	10

$\sqrt{x}$	2	3	4	5
$2 \sqrt{x}$	4	6	8	10

So it follows that:

$$y = 2 \sqrt{x}$$